OCR
RECOGNISING ACHIEVEMENT

GCE

Mathematics

Advanced GCE

Unit **4725:** Further Pure Mathematics 1

Mark Scheme for January 2012

Oxford Cambridge and RSA Examinations

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of candidates of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, OCR Nationals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support, which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

Any enquiries about publications should be addressed to:

OCR Publications
PO Box 5050
Annesley
NOTTINGHAM
NG15 0DL

Telephone: 0870 770 6622
Facsimile: 01223 552610
E-mail: publications@ocr.org.uk

Annotations

Annotation in scoris	Meaning
✓and ✖	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	

Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awar marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark schem assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work mus the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the wo looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect m must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, awa the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involve your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candida intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem i substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or ex… th… of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct… ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to … candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the schem… otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a part… dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate… wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the… two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be …

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously inco… Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accura… given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage… may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in th… rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will… through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is … image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-…

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates oth… are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the… variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) sh… penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The … any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme… contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then exa… the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears…

(complete) attempt and ignore the others.
NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is gene this may differ for some units. This is achieved by withholding one A or B mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guida
1			$a^2 + 5^2 = 13^2$ $a = 12$ $\tan^{-1}\frac{5}{a}$ 0.395 or 22.6° or 0.126π	M1 A1 M1 A1FT **[4]**	Use formula for modulus Obtain correct answer Use formula for argument Obtain correct answer allow 0.39	
2			$3p + 4q = 1, \quad -3p - 5q = 1, \quad 2p + 3q = 0$ $p = 3$ and $q = -2$	B1 M1 A1 M1 A1 **[5]**	State identity matrix is $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ Find a pair of simultaneous equations Correct pair of distinct equations Attempt to solve Obtain correct answers	
3			$x^2 - y^2 = 3$ and $xy = 3\sqrt{2}$ $x^4 - 3x^2 - 18 = 0$ or $y^4 + 3y^2 - 18 = 0$ $x = \pm\sqrt{6}$ or $y = \pm\sqrt{3}$ $\pm(\sqrt{6} + i\sqrt{3})$	M1 A1 M1 M1 A1 A1 **[6]**	Attempt to equate real and imaginary parts Obtain both results Eliminate to obtain quadratic in x^2 or y^2 Solve to obtain x or y value Both values correct Correct answers as complex numbers	

Question			Answer	Marks	Guida	
4			$\frac{1}{4}n^2(n+1)^2 - \frac{3}{2}n(n+1)$ $\frac{1}{4}n(n+1)(n+3)(n-2)$	M1 DM1 A1 M1 A1 A1 **[6]**	Express as difference of two series Use standard series results Obtain correct unsimplified answer Attempt to factorise At least factor of $n(n+1)$ Obtain correct answer	From their u
5	(a)		$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$	B1 B1 **[2]**	Each column correct	
5	(b)	(i)		B1 DB1 **[2]**	Stretch Scale factor 4 in the y direction	Not "in the
5	(b)	(ii)	4	B1 B1 **[2]**	Correct value of determinant Scale factor for area	Allow scale
6				B1 B1 B1 B1 B1 B1 **[6]**	Circle Centre $(\sqrt{3},1)$ Passing through O and crosses y-axis again Line, with correct slope shown $\frac{1}{2}$ line starting at O Completely correct diagram for both loci	Ignore shad

Question			Answer	Marks	Guida	
7	(i)			M1 A1 A1 **[3]**	Attempt at matrix multiplication Obtain $\mathbf{M}^2$ correctly Obtain **given answer** correctly	
7	(ii)		$\begin{pmatrix} 3^n & 0 \\ 3^n - 1 & 1 \end{pmatrix}$	B1 B1 **[2]**	3 elements correct 4^{th} element correct	
7	(iii)		$\begin{pmatrix} 3^{k+1} & 0 \\ 3^{k+1} - 1 & 1 \end{pmatrix}$	B1 M1 A1 B1 **[4]**	Show that their result is true for $n = 1$ or 2 Attempt to find $\mathbf{M}^k \cdot \mathbf{M}$ or vice versa Obtain correct answer Complete statement of induction conclusion	Must have
8	(i)			M1 A1 **[2]**	Combine with a common denominator Obtain **given** answer correctly	
8	(ii)		$\frac{n}{n+1}$	M1 A1 M1 A1 **[4]**	Express terms using **(i)** At least 1^{st} two and last two correct Show terms cancelling Obtain correct answer, in terms of n	

Question			Answer	Marks	Guida
8	(iii)		$1-\dfrac{n}{n+1}$	B1 B1FT **[2]**	$\lim\limits_{n\to\infty}\dfrac{n}{n+1}=1$ This value – **(ii)**
9	(i)		det **X** = $\Delta = 10-9a-a^2$	M1 M1 A1 **[3]**	Show correct expansion process for 3×3 Correct evaluation of any 2×2 Obtain correct answer aef
9	(ii)		$a = 1$ or -10	M1 A1FT A1FT **[3]**	Their det **X** = 0 Obtain correct answers from their **(i)**
9	(iii)		$\frac{1}{\Delta}\begin{pmatrix} -a & 2 & 6-9a \\ 5 & -a-9 & 18-3a \\ -a & 2 & a^2-4 \end{pmatrix}$	M1 A1 A1 B1ft **[4]**	Show correct process for adjoint entries Obtain at least four correct entries in adjoint Obtain completely correct adjoint Divide by their determinant
10	(i)		$\alpha+\beta+\gamma=3$ $\alpha\beta+\beta\gamma+\gamma\alpha=2$ $\alpha\beta\gamma=-\frac{2}{3}$	B1 B1 B1 **[3]**	State correct value State correct value State correct value

Question			Answer	Marks	Guida
10	(ii)		EITHER		
				M1	$c = (\pm)\alpha^2\beta^2\gamma^2$
			$c = -\frac{4}{9}$	A1FT	Obtain **given** correct answer
			$\sum\alpha^2 = (\sum\alpha)^2 - 2\sum\alpha\beta$	M1	Use correct expression
			5	A1FT	Obtain correct value
			$a = -5$	A1FT	Obtain answer correctly
				M1*	Attempt to find an identity
			$\sum\alpha^2\beta^2 = (\sum\alpha\beta)^2 - 2\alpha\beta\gamma\sum\alpha$	A1	Obtain correct identity
				DM1	Use appropriate values
			$b = 8$	A1	Obtain correct answer cao
				[9]	
			OR		
				B1	State or use correct substitution
				M1	Rearrange, fractional indices isolated
				DM1	Square both sides
				DM1	Expand and simplify
			$9y^3 - 45y^2 + 72y - 4 = 0$	A1	Obtain correct equation
				M1	Use coefficients of their cubic
			$c = -\frac{4}{9}$	A1	Obtain given answer correctly
			$a = -5$	A1FT	Obtain correct answer
			$b = 8$	A1FT	Obtain correct answer
				[9]	**SC** mixture of methods only A1FT for a and b

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
Cambridge
CB1 2EU

OCR Customer Contact Centre

Education and Learning
Telephone: 01223 553998
Facsimile: 01223 552627
Email: general.qualifications@ocr.org.uk

www.ocr.org.uk

For staff training purposes and as part of our quality assurance programme your call may be recorded or monitored

Oxford Cambridge and RSA Examinations
is a Company Limited by Guarantee
Registered in England
Registered Office; 1 Hills Road, Cambridge, CB1 2EU
Registered Company Number: 3484466
OCR is an exempt Charity

OCR (Oxford Cambridge and RSA Examinations)
Head office
Telephone: 01223 552552
Facsimile: 01223 552553